PREFACE

Calculus is one of the greatest achievements of the human intellect. Inspired by problems in astronomy, Newton and Leibniz developed the ideas of calculus 300 years ago. Since then, each century has demonstrated the power of calculus to illuminate questions in mathematics, the physical sciences, engineering, and the social and biological sciences.

Calculus has been so successful because of its extraordinary power to reduce complicated problems to simple rules and procedures. Therein lies the danger in teaching calculus: it is possible to teach the subject as nothing but the rules and procedures—thereby losing sight of both the mathematics and of its practical value. This edition of *Calculus* continues our effort to refocus the teaching of calculus on concepts as well as procedures.

A Focused Vision: Conceptual Understanding

Our goal is to provide students with a clear understanding of the ideas of calculus as a solid foundation for subsequent courses in mathematics and other disciplines. When we designed this curriculum we started with a clean slate. We increased the emphasis on some topics, such as differential equations, and decreased the emphasis on others after discussions with mathematicians, engineers, physicists, chemists, biologists, and economists. We focused on key concepts, emphasizing depth of understanding rather than breadth of coverage.

The Fourth Edition: Flexible and Focused

Since our First Edition struck a new balance between concepts, modeling, and skills in 1992, we have helped widen the range of choices for calculus instructors. Under our approach, which we called the "Rule of Four," ideas are presented graphically, numerically, symbolically, and verbally. Our problems probe student understanding in areas often taken for granted. The influence of these problems, praised for their creativity and variety, has extended far beyond the users of our textbook.

The text retains the hallmarks of our earlier editions: The Rule of Four, an emphasis on modeling, exposition that students can read, and a flexible approach to technology. Working in collaboration, consortium members and colleagues in mathematics and client disciplines have shaped a flexible curriculum that balances new and traditional topics.

In adding new topics, we have kept in mind two competing considerations: a textbook's role as a reference suitable for differing audiences, and the danger of encyclopedic clutter that overwhelms students and obscures the main vista. Our treatment of new topics is guided by the Rule of Four and our focus on conceptual understanding.

Choice of Paths: Lean or Expanded

For those who prefer the lean topic list of earlier editions, we have kept clear the main conceptual paths. For example,

- The Key Concept chapters on the derivative and the definite integral (Chapters 2 and 5) can be covered at the outset of the course, right after Chapter 1.
- Limits and Continuity (Sections 1.7 and 1.8) can be covered in depth before the introduction of the derivative (Sections 2.1 and 2.2), or after.
- Approximating Functions Using Sefies (Chapter 10) can be covered before, or without, Chapter 9.
- In Chapter 4 (Using the Derivative), instructors can select freely from Sections 4.4-4.8.
- Chapter 8 (Using the Definite Integral) contains a wide range of applications. Instructors can select one or two to do in detail.

The Development of Mathematical Thinking

The first stage in the development of mathematical thinking is the acquisition of a clear intuitive picture of the central ideas. In the next stage, the student learns to reason with the intuitive ideas and explain the reasoning clearly in plain English. After this foundation has been laid, there is a choice of direction. All students benefit from both theory and modeling, but the balance may differ for different groups. Some students, such as mathematics majors, may prefer more theory, while others, such as science and engineering majors, may prefer more modeling.

The Development of Mathematical Skills

To use calculus effectively, students need skill in both symbolic manipulation and the use of technology. The exact proportions of each may vary widely, depending on the preparation of the student and the wishes of the instructor. The book is adaptable to many different combinations.

The book does not require any specific software or technology. It has been used with graphing calculators, graphing software, and computer algebra systems. Any technology with the ability to graph functions and perform numerical integration will suffice. Students are expected to use their own judgment to determine where technology is useful.

Chapter-by-Chapter Description of Changes in the Fourth Edition:

Throughout the chapters, problems and exercises have been reorganized and hundreds of new problems and exercises have been added.

- Chapter 1: The chapter now ends with a section on limits, formerly Section 2.2. This allows a fuller discussion of continuity, both on an interval and at a point.
- Chapter 2: Some material has been reorganized. The material on limits and the discussion of continuity formerly in Section 2.7 has been moved to the end of Chapter 1.
- Chapter 3: There is a new section applying the chain rule to the derivatives of inverse functions. The section on hyperbolic functions has been moved to this chapter from Chapter 4. Section 3.3 on the derivatives of exponential functions has been extensively rewritten. The section on the chain rule has been expanded. Most of the theory on differentiable functions, including the Mean Value Theorem, has been moved from Section 4.7 to the end of this chapter. The second order error behavior of the tangent line approximation is addressed in greater detail, as a preview to Taylor polynomials.
- Chapter 4: There is a new section on rates and related rates, with dozens of new problems and exercises. The treatment of l'Hopital's rule has been moved from Chapter 3 to this chapter and expanded. The material on parametric equations has been moved to this chapter from Chapter 3 and expanded to included concavity of parametric curves.
- Chapter 5: This chapter has been reorganized and rewritten to clarify the Fundamental Theorem of Calculus, viewed as the integral of a rate of change giving the total change. There is also new material on area between curves.
- Chapter 6: There are no major changes in this chapter.
- Chapter 7: There are no major changes in this chapter.
- Chapter 8: There is a new section on area, arc length, and slope for curves in polar coordinates, including the former Appendix B on polar coordinates. Many of the problems and exercises in this section are new.
- Chapter 9: The chapter begins with a new section on sequences, including recursively defined sequences and a more detailed discussion of convergence for bounded sequences. Nearly all the problems and exercises in this section are new. The role of p-series in the integral comparison test is delineated. In Section 9.4, the limit comparison test has been added to the array of tests for convergence of a series, and there is a discussion of conditional convergence. The last section on functions defined by power series has been expanded, with more analysis of behavior at the endpoints of the interval of convergence. There is a new project proving the convergence of the sequence $s_n = (1 + 1/n)^n$, using the Bounded Monotone Convergence Theorem.

- Chapter 10: Section 10.4 on the Lagrange error bound for Taylor polynomials has been revised.
- Chapter 11: There is a new project on the 2003 SARS outbreak.
- Appendix: Polar Coordinates have been moved to Chapter 8.

Content

This content represents our vision of how calculus can be taught. It is flexible enough to accommodate individual course needs and requirements. Topics can easily be added or deleted, or the order changed.

Chapter 1: A Library of Functions

This chapter introduces all the elementary functions to be used in the book. Although the functions are probably familiar, the graphical, numerical, verbal, and modeling approach to them may be new. We introduce exponential functions at the earliest possible stage, since they are fundamental to the understanding of real-world processes. The chapter concludes with sections on limits and continuity.

Chapter 2: Key Concept—The Derivative

The purpose of this chapter is to give the student a practical understanding of the definition of the derivative and its interpretation as an instantaneous rate of change. The power rule is introduced; other rules are introduced in Chapter 3.

Chapter 3: Short-Cuts to Differentiation

The derivatives of all the functions in Chapter 1 are introduced, as well as the rules for differentiating products, quotients, composite, inverse, hyperbolic, and implicitly defined functions.

Chapter 4: Using the Derivative

The aim of this chapter is to enable the student to use the derivative in solving problems, including optimization, graphing, rates, parametric equations, and indeterminate forms. It is not necessary to cover all the sections in this chapter.

Chapter 5: Key Concept—The Definite Integral

The purpose of this chapter is to give the student a practical understanding of the definite integral as a limit of Riemann sums and to bring out the connection between the derivative and the definite integral in the Fundamental Theorem of Calculus.

Chapter 6: Constructing Antiderivatives

This chapter focuses on going backward from a derivative to the original function, first graphically and numerically, then analytically. It introduces the Second Fundamental Theorem of Calculus and the concept of a differential equation.

Chapter 7: Integration

This chapter includes several techniques of integration, including substitution, parts, partial fractions, and trigonometric substitutions; others are included in the table of integrals. There are discussions of numerical methods and of improper integrals.

Chapter 8: Using the Definite Integral

This chapter emphasizes the idea of subdividing a quantity to produce Riemann sums which, in the limit, yield a definite integral. It shows how the integral is used in geometry, physics, economics, and probability; polar coordinates are introduced. It is not necessary to cover all the sections in this chapter.

Chapter 9: Sequences and Series

This chapter focuses on sequences, series of constants and convergence. It includes the integral, ratio, comparison, limit comparison, and alternating series tests. It also introduces geometric series and general power series, including their intervals of convergence.

Chapter 10: Approximating Functions

This chapter introduces Taylor Series and Fourier Series using the idea of approximating functions by simpler functions.

Chapter 11: Differential Equations

This chapter introduces differential equations. The emphasis is on qualitative solutions, modeling, and interpretation.

Appendices

There are appendices on roots, accuracy, and bounds; complex numbers; and Newton's Method.

Supplementary Materials and Additional Resources

Supplements for the instructor can be obtained by sending a request on your institutional letterhead to Mathematics Marketing Manager, John Wiley & Sons, Inc., 111 River Street, Hoboken, NJ 07030, or by contacting your Wiley representative. The following supplementary materials are available for this edition:

- **Instructor's Manual** containing teaching tips, calculator programs, overhead transparency masters.
- **Test Bank** containing test questions and solutions arranged by section.
- **Instructor's Solution Manual** with complete solutions to all problems.
- **Student's Solution Manual** with complete solutions to half the odd-numbered problems.
- **Student's Study Guide** with additional study aids for students that are tied directly to the book.
- **Additional Material,** elaborating specially marked points in the text, as well as password protected electronic versions of the instructor ancillaries, can be found on the web at www.wiley.com/college/hugheshallett.

ConcepTests

ConcepTests, modeled on the pioneering work of Harvard physicist Eric Mazur, are questions designed to promote active learning during class, particularly (but not exclusively) in large lectures. Our evaluation data show students taught with ConcepTest outperformed students taught by traditional lecture methods 73% versus 17% on conceptual questions, and 63% versus 54% on computational problems. ConcepTests arranged by section are available from your Wiley representative.

Faculty Resource Network

A peer-to-peer network of academic faculty dedicated to the effective use of technology in the classroom, this group can help you apply innovative classroom techniques and implement specific software packages. Visit www.facultyresourcenetwork.com or speak to your Wiley representative.

eGrade Plus

eGrade Plus is a powerful online tool that provides a completely integrated suite of teaching and learning resources in one easy-to-use Web site. *eGrade Plus* includes an online version of the text, with fully integrated electronic versions of all student supplements, including the Student Solutions Manual, the Student Study Guide, and an Algebra and Trigonometry Refresher. Instructors will have additional access to electronic versions of the Instructor's Manual, the Instructor's Solutions Manual, additional projects, as well as other valuable resources. *eGrade Plus* **also** offers an online assessment system with full gradebook capabilities, which contains over 1000 skill-building questions from the Exercise sections in each chapter. Please view our online demo at www.wiley.com/college/egradeplus. Here you will find additional information about the features and benefits of eGrade Plus, how to request a "test drive" of eGrade Plus, and how to adopt it for class use.

AP Teacher's Guide

The AP Guide, written by experienced AP teachers, provides day-by-day syllabi for AB and BC Calculus, sample multiple choice questions, a listing of the past 25 years of AP free-response questions by chapter of the text, teaching tips, and labs to encourage student exploration of concepts.

Acknowledgements

First and foremost, we want to express our appreciation to the National Science Foundation for their faith in our ability to produce a revitalized calculus curriculum and, in particular, to our program officers, Louise Raphael, John Kenelly, John Bradley, and James Lightbourne. We also want to thank the members of our Advisory Board, Benita Albert, Lida Barrett, Simon Bemau, Robert Davis, M. Lavinia DeConge-Watson, John Dossey, Ron Douglas, Eli Fromm, William Haver, Seymour Parter, John Prados, and Stephen Rodi.

In addition, a host of other people around the country and abroad deserve our thanks for their contributions to shaping this edition. They include: Huriye Arikan, Ralph Baierlein, Ruth Baruth, Frank Beatrous, Alon Ben-David, Ebo Bentil, Paul Blanchard, Michael Boezi, Kelly Boyle, Melkana Brakalova, David Bressoud, Stephen Boyd, Morton Brown, Lucille Buonocore, Rick Cangelosi, Jo Cannon, Ray Cannon, Marilyn Carlson, Phil Cheifetz, C.K. Cheung, Scott Clark, Eric Connally, Caspar Curjel, Ehud de Shalit, Ann Davidian, Bob Decker, Dennis De Turck, Tom Dick, Srdjan Divac, Tevian Dray, Jeff Edmunds, Wade Ellis, Johann Engelbrecht, Bill Faris, Paul Feehan, Hermann Flaschka, Sol Friedberg, Paul Froeschl, Amanda Galtman, Lynn Garner, Tom Gearhart, Chris Goff, Joshua Green, Cristi Guevara, John Hagood, Gregory Hartman, Dudley Herschbach, Rob Indik, Adrian Iovita, Pallavi Jayawant, Millie Johnson, Alex Kasman, Matthias Kawski, Mike Klucznik, Donna Krawczyk, Mark Kunka, Ted Laetsch, Stephane Lafortune, Brigitte Lahme, Carl Leinbach, Carl Leinert, David Levermore, Don Lewis, Jim Lewis, Julie Lindstrom, Guadalupe Lozano, Bin Lu, Chris Lucas, John Lucas, Dan Madden, Alex Mallozzi, Corinne Manogue, Lisa A. Mantini, Nancy Marcus, Eric Mazur, Dan McGee, Tom McMahon, Ansie Meiring, Dave Meredith, Andrew Metrick, Lang Moore, Jerry Morris, Alan Newell, Steve Olson, John Orr, Arnie Ostebee, Igor Padure, Howard Penn, Amy Radunskaya, Wayne Raskind, Jeffrey Rauch, Janet Ray, Maria Robinson, Laurie Rosatone, Tenzin Chokey Rubling, Ann Ryu, Ayse Sahin, Nataliya Sandler, Ken Santor, Anne Scanlan-Rohrer, Pat Shure, Scott Pilzer, Lynne Small, David Smith, Seung Hye Song, Bob Speiser, Howard Stone, Steve Strogatz, "Suds" Sudholz, Al Taylor, Peter Taylor, Ralph Teixeira, Sally Thomas, Alan Tucker, Jerry Uhl, Doug Ulmer, Ignatios Vakalis, Bill Velez, Aaron Wootton, Xianbao Xu, Kathy Yoshiwara, Lee Zia, and Paul Zorn.

Reports from the following reviewers were most helpful for the third edition:

Jennifer Aikens, Dean Allison, Cathy Anderson, Richard I. Avery, Janet Beery, Hamid Behmard, Victor Belli, Tom Bengtson , David Borthwick, Sean Bradley, Carl T. Brezovec, Paul Wayne Brill, Denise Brown, Dave Bush, Don A. Campbell, Jack Carter, Stan Chadick, Ruth Charney, Jon Clauss, Ann Commito, Lawerence J. Cohen, Mary V. Connolly, Randall Crist, Rick Dalrymple, John Davis, Mike Dellens, Matt DeLong, Sylvia Desjardins, Meighan I. Dillon, Dawn Draus, Steven R. Dunbar, Jim Edmondson, Karen Edwards, Murray Eisenberg, Sue Esch, Phil Farmer, Richard E. Faulkenberry, George F. Feissner, Nelson Fong, Paula Castagna, Ben Fusaro, Kris Green, Michael Green, Leah Griffith, Cheryl Grood, David L. Gross, Brenda Guthrie, John Hagood, William Hamill, John Haverhals, Elizabeth Hentges, Curtis Herink, Jim Holmes, John Hoover, Rick Hough, Thomas Hunter, Kathy Ivey, Craig A. Jensen, Sue Jensen, Jerry Johnson, Nancy L. Johnson, Guarav Khanna, Helmut Knaust, Robert Kowalczyk, Anne Landry, James W. Lea, Phyllis Leonard, Tom Linton, Harold Lund, Juan Manfredi, William O. Martin, Alex Martsinkovsky, Anita Maxwell, Barbara McGee, Mary Ann McLoughlin, Douglas B. Meade, Richard Mercer, Raquel Mesa, C.R. Messer, Mark A. Mills, Yair Minsky, Derek Mitchem, Jeff Mock, Doug Mupasiri, Douglas Nelson, Dave Nolan, Sandy Norman, David A. Olson, Amos Ong, Grace Orzech, Wes Ostertag, Jeffrey S. Palmer, Barry Perratt, Peter Phillíou, Shirley Pomeranz, G.J. Porter, Rebecca Porter, Vrunda Prabhu, Harald Proppe, Margie Reinhart, Mike Robillard, Donna Fields Rochon, Peter Rojas, Ned I. Rosen, Peler Rosnick, Tom Rousseau, Brenda Rudin, Mark D. Schlatter, Howard Schwesky, Patrick Shanahan, Michael Sheard, Angela B. Shiflet, Mehrdad Simkani, Lynne Small, Katy Snyder, Edward J. Soares, Ernie Solheid, Richard Stout, Robert Styer, James Sullivan, Ted Sundstrom, Stuart Swain, Ray Tennant, Buren Thomas, George Tintera, Dave Trautman, Mark Turner, Marian Van Vleet, Mike Veatch, Rachel Westlake, Bill Waltmann, Michael Ward, Steve Waters, Jack Winn, John Wood, Lisa Woodrum, William Wunderlich, Christos Xenophontos, W. Xie, Deane Yang, Kathie

Yoder, Bruce Yoshiwara, Andrzej Zarach, Ed Zeidman, and Peter Zekert.

Reports from the following reviewers were most helpful for the fourth edition:

Patricia Beaulieu, Phil Blau, Russell D. Blyth, Ray Cannon, Paula Castagna, Annette Cooper, Randall Crist, James D'Archangelo, Michael Divinia, Diane Doyle, Jim Edmondson, Leslie O. Foged, Scott R. Fulton, Lynn E. Garner, Sandy Grabiner, Steve Harris, Jeff Hirst, Vlajko Kocic, Kenneth Levasseur, Greg Marks, Kevin McDougal, Wilbur Mellema, Dan Oberlin, Tom Rousseau, Jeffrey Scroggs, Ernie S. Solheid, Barry S. Spieler, Diana Staats, Steve Szydlik, Mark Turner, Tony Vance, Gordon Woodward, and William Wunderlich.

Deborah Hughes-Hallett	Patti Frazer Lock	Douglas Quinney
Andrew M. Gleason	David O. Lomen	Karen R. Rhea
William G. McCallum	David Lovelock	Jeff Tecosky-Feldman
Daniel E. Flath	Brad G. Osgood	Thomas W. Tucker

To Students: How to Learn from this Book

- This book may be different from other math textbooks that you have used, so it may be helpful to know about some of the differences in advance. This book emphasizes at every stage the *meaning* (in practical, graphical or numerical terms) of the symbols you are using. There is much less emphasis on "plug-and-chug" and using formulas, and much more emphasis on the interpretation of these formulas than you may expect. You will often be asked to explain your ideas in words or to explain an answer using graphs.
- The book contains the main ideas of calculus in plain English. Your success in using this book will depend on your reading, questioning, and thinking hard about the ideas presented. Although you may not have done this with other books, you should plan on reading the text in detail, not just the worked examples.
- There are very few examples in the text that are exactly like the homework problems. This means that you can't just look at a homework problem and search for a similar-looking "worked out" example. Success with the homework will come by grappling with the ideas of calculus.
- Many of the problems that we have included in the book are open-ended. This means that there may be more than one approach and more than one solution, depending on your analysis. Many times, solving a problem relies on common sense ideas that are not stated in the problem but which you will know from everyday life.
- Some problems in this book assume that you have access to a graphing calculator or computer. There are many situations where you may not be able to find an exact solution to a problem, but you can use a calculator or computer to get a reasonable approximation.
- This book attempts to give equal weight to four methods for describing functions: graphical (a picture), numerical (a table of values) algebraic (a formula), and verbal. Sometimes you may find it easier to translate a problem given in one form into another. The best idea is to be flexible about your approach: if one way of looking at a problem doesn't work, try another.
- Students using this book have found discussing these problems in small groups very helpful. There are a great many problems which are not cut-and-dried; it can help to attack them with the other perspectives your colleagues can provide. If group work is not feasible, see if your instructor can organize a discussion session in which additional problems can be worked on.
- You are probably wondering what you'll get from the book. The answer is, if you put in a solid effort, you will get a real understanding of one of the most important accomplishments of the millennium—calculus—as well as a real sense of the power of mathematics in the age of technology.