1. Tire Characteristics and Vehicle Handling and Stability
   1.1. Introduction
   1.2. Tire and Axle Characteristics
       1.2.1. Introduction to Tire Characteristics
       1.2.2. Effective Axle Cornering Characteristics
   1.3. Vehicle Handling and Stability
       1.3.1. Differential Equations for Plane Vehicle Motions
       1.3.2. Linear Analysis of the Two-Degree-of-Freedom Model
       1.3.3. Nonlinear Steady-State Cornering Solutions
       1.3.4. The Vehicle at Braking or Driving
       1.3.5. The Moment Method
       1.3.6. The Car-Trailer Combination
       1.3.7. Vehicle Dynamics at More Complex Tire Slip Conditions

2. Basic Tire Modeling Considerations
   2.1. Introduction
   2.2. Definition of Tire Input Quantities
   2.3. Assessment of Tire Input Motion Components
   2.4. Fundamental Differential Equations for a Rolling and Slipping Body
   2.5. Tire Models (Introductory Discussion)

3. Theory of Steady-State Slip Force and Moment Generation
   3.1. Introduction
   3.2. Tire Brush Model
       3.2.1. Pure Side Slip
       3.2.2. Pure Longitudinal Slip
       3.2.3. Interaction between Lateral and Longitudinal Slip (Combined Slip)
       3.2.4. Camber and Turning (Spin)
3.3. The Tread Simulation Model 128
3.4. Application: Vehicle Stability at Braking up to Wheel Lock 140

4. Semi-Empirical Tire Models 150
4.1. Introduction 150
4.2. The Similarity Method 152
4.2.1. Pure Slip Conditions 152
4.2.2. Combined Slip Conditions 158
4.2.3. Combined Slip Conditions with $F_s$ as Input Variable 163
4.3. The Magic Formula Tire Model 165
4.3.1. Model Description 165
4.3.2. Full Set of Equations 176
4.3.3. Extension of the Model for Turn Slip 183
4.3.4. Ply-Steer and Conicity 191
4.3.5. The Overturning Couple 196
4.3.6. Comparison with Experimental Data for a Car, a Truck, and a Motorcycle Tire 202

5. Non-Steady-State Out-of-Plane String-Based Tire Models 212
5.1. Introduction 212
5.2. Review of Earlier Research 215
5.3. The Stretched String Model 216
5.3.1. Model Development 225
5.3.2. Step and Steady-State Response of the String Model 232
5.3.3. Frequency Response Functions of the String Model 232
5.4. Approximations and Other Models 240
5.4.1. Approximate Models 241
5.4.2. Other Models 256
5.4.3. Enhanced String Model with Tread Elements 258
5.5. Tire Inertia Effects 268
5.5.1. First Approximation of Dynamic Influence (Gyroscopic Couple) 269
5.5.2. Second Approximation of Dynamic Influence (First Harmonic) 271
5.6. Side Force Response to Time-Varying Load 277
5.6.1. String Model with Tread Elements Subjected to Load Variations 277
5.6.2. Adapted Bare String Model 281
5.6.3. The Force and Moment Response 284

6. Theory of the Wheel Shimmy Phenomenon 287
6.1. Introduction 287
6.2. The Simple Trailing Wheel System with Yaw Degree of Freedom 288

7. Single-Contact-Point Models 298
7.1. Introduction 298
7.2. Model Development 301
7.2.1. Linear Models 301
7.2.2. Semi-Nonlinear Models 308
7.2.3. Fully Nonlinear Models 314
7.2.4. Nonlagging Models 318
7.2.5. The Gyroscopic Effect 320
7.3. Enhanced Nonlinear Models 326

8. Applications of Tire Models 333
8.1. Vehicle Response 333
8.2. Cornering on Uneven Roads 344
8.3. Longitudinal Force and Axle Motions, and ABS braking 364
8.3.1. Effective Rotating Moment 364
8.3.2. Computation of Force Response Functions 371
8.3.3. Frequency Response 371
8.3.4. Frequency Response Functions 375
8.4. Forced Steering and ABS Braking 383
8.4.1. Dynamics of ABS Braking by Wheel Locking 383
8.4.2. Dynamics of ABS Braking with Tire Properties 395
8.5. ABS Braking on Uneven Roads 410
8.5.1. In-Plane Motion Assembly 410
8.5.2. Antilock Braking System Assembly 415
8.6. Starting from Static Equilibrium 421

9. Short Wavelength Tire Model 432
9.1. Introduction 432
## Contents

6.3. Systems with Yaw and Lateral Degrees of Freedom 295
6.3.1. Yaw and Lateral Degrees of Freedom with Rigid Wheel/Tire (Third Order) 296
6.3.2. The Fifth-Order System 297
6.4. Shimmy and Energy Flow 311
6.4.1. Unstable Modes and the Energy Circle 311
6.4.2. Transformation of Forward Motion Energy into Shimmy Energy 317
6.5. Nonlinear Shimmy Oscillations 320

7. Single-Contact-Point Transient Tire Models
7.1. Introduction 329
7.2. Model Development
7.2.1. Linear Model 330
7.2.2. Semi-Non-Linear Model 335
7.2.3. Fully Nonlinear Model 336
7.2.4. Nonlagging Part 345
7.2.5. The Gyroscopic Couple 348
7.3. Enhanced Nonlinear Transient Tire Model 349

8. Applications of Transient Tire Models
8.1. Vehicle Response to Steer Angle Variations 356
8.2. Cornering on Undulated Roads 356
8.3. Longitudinal Force Response to Tire Nonuniformity, Axle Motions, and Road Unevenness 366
8.3.1. Effective Rolling Radius Variations at Free Rolling 367
8.3.2. Computation of the Horizontal Longitudinal Force Response 371
8.3.3. Frequency Response to Vertical Axle Motions 374
8.3.4. Frequency Response to Radial Run-out 376
8.4. Forced Steering Vibrations 379
8.4.1. Dynamics of the Unloaded System Excited by Wheel Unbalance 380
8.4.2. Dynamics of the Loaded System with Tire Properties Included 382
8.5. ABS Braking on Undulated Road 385
8.5.1. In-Plane Model of Suspension and Wheel/Tire Assembly 386
8.5.2. Antilock Braking Algorithm and Simulation 390
8.6. Starting from Standstill 394

9. Short Wavelength Intermediate Frequency Tire Model
9.1. Introduction 404
## 9. Contact Patch Slip Model
9.2. The Contact Patch Slip Model
9.2.1. Brush Model Non-Steady-State Behavior
9.2.2. The Model Adapted to the Use of the Magic Formula
9.2.3. Parking Maneuvers

## 9. Tire Dynamics
9.3. Dynamic Tire Model Performance
9.3.1. Dynamic Equations
9.3.2. Constitutive Relations

## 10. Dynamic Tire Response to Short Road Unevennesses
10.1. Model Development
10.1.1. Tire Envelopment Properties
10.1.2. The Effective Road Plane Using Basic Functions
10.1.3. The Effective Road Plane Using the 'Cam' Road
10.1.4. The Effective Rolling Radius When Rolling Over a Cleat
10.1.5. The Location of the Effective Road Plane
10.2. SWIFT on Road Unevennesses (Simulation and Experiment)
10.2.1. Two-Dimensional Unevennesses
10.2.2. Three-Dimensional Unevennesses

## 11. Motorcycle Dynamics
11.1. Introduction
11.2. Model Description
11.2.1. Geometry and Inertia
11.2.2. The Steer, Camber, and Slip Angles
11.2.3. Air Drag, Driving or Braking, and Fore-and-Aft Load Transfer
11.2.4. Tire Force and Moment Response
11.3. Linear Equations of Motion
11.3.1. The Kinetic Energy
11.3.2. The Potential Energy and the Dissipation Function
11.3.3. The Virtual Work
11.3.4. Complete Set of Linear Differential Equations
11.4. Stability Analysis and Step Responses
11.4.1. Free Uncontrolled Motion
11.4.2. Step Responses of Controlled Motion
11.5. Analysis of Steady-State Cornering
11.5.1. Linear Steady-State Theory
11.5.2. Non-Linear Analysis of Steady-State Cornering
11.5.3. Modes of Vibration at Large Lateral Accelerations
11.6. The Magic Formula Tire Model

## 12. Tire Steady-State and Non-Linear Analysis

## 13. Outlines of Three Additional Chapters
13.1. The RMOD-K Tire Model
13.1.1. The Nonlinear RMOD-K Model
13.1.2. The Flexible RMOD-K Model
13.1.3. Comparison of Models
13.2. The FTire Tire Model
13.2.1. Introduction
13.2.2. Structure Model
13.2.3. Tread Model
13.2.4. Model Data
13.3. The MF-Swift Tire Model
13.3.1. Introduction
13.3.2. Model Overview
13.3.3. MF-Tire/MF-Swift Parameters
13.3.4. Test and Model Comparison

## References

## List of Symbols

## Appendix
1. Sign Conventions for Dynamic Tire Models
2. Online Information
3. MF-Tire/MF-Swift Parameters