Contents

Notation xvi

Part I: Gravitational-wave theory 1

1 The geometric approach to GWs 3
1.1 Expansion around flat space 4
1.2 The transverse-traceless gauge 7
1.3 Interaction of GWs with test masses 13
1.3.1 Geodesic equation and geodesic deviation 13
1.3.2 Local inertial frames and freely falling frames 15
1.3.3 TT frame and proper detector frame 17
1.4 The energy of GWs 26
1.4.1 Separation of GWs from the background 27
1.4.2 How GWs curve the background 29
1.4.3 The energy–momentum tensor of GWs 35
1.5 Propagation in curved space-time 40
1.5.1 Geometric optics in curved space 42
1.5.2 Absorption and scattering of GWs 46
1.6 Solved problems 48
1.1. Linearization of the Riemann tensor in curved space 48
1.2. Gauge transformation of $h_{\mu\nu}$ and $R^{(1)}_{\mu\nu\rho\sigma}$ 49
Further reading 51

2 The field-theoretical approach to GWs 52
2.1 Linearized gravity as a classical field theory 53
2.1.1 Noether's theorem 53
2.1.2 The energy–momentum tensor of GWs 58
2.1.3 The angular momentum of GWs 61
2.2 Gravitons 66
2.2.1 Why a spin-2 field? 66
2.2.2 The Pauli–Fierz action 70
2.2.3 From gravitons to gravity 74
2.2.4 Effective field theories and the Planck scale 79
2.3 Massive gravitons 81
2.3.1 Phenomenological bounds 82
2.3.2 Field theory of massive gravitons 84
2.4 Solved problems 95
2.1. The helicity of gravitons 95
2.2. Angular momentum and parity of graviton states 98
Further reading 100

3 Generation of GWs in linearized theory 101
3.1 Weak-field sources with arbitrary velocity 102
3.2 Low-velocity expansion 105
3.3 Mass quadrupole radiation 109
3.3.1 Amplitude and angular distribution 109
3.3.2 Radiated energy 113
3.3.3 Radiated angular momentum 114
3.3.4 Radiation reaction on non-relativistic sources 116
3.3.5 Radiation from a closed system of point masses 121
3.4 Mass octupole and current quadrupole 125
3.5 Systematic multipole expansion 131
3.5.1 Symmetric-trace-free (STF) form 134
3.5.2 Spherical tensor form 139
3.6 Solved problems 156
3.1. Quadrupole radiation from an oscillating mass 156
3.2. Quadrupole radiation from a mass in circular orbit 158
3.3. Mass octupole and current quadrupole radiation from a mass in circular orbit 161
3.4. Decomposition of $\dot{S}^{kl,m}$ into irreducible representations of $SO(3)$ 163
3.5. Computation of $\int d\Omega\, (\mathbf{T}^{E2,B2}_{lm})^*_{ij} n_{i_1} \cdots n_{i_\alpha}$ 165
Further reading 166

4 Applications 167
4.1 Inspiral of compact binaries 167
4.1.1 Circular orbits. The chirp amplitude 169
4.1.2 Elliptic orbits. (I) Total power and frequency spectrum of the radiation emitted 176
4.1.3 Elliptic orbits. (II) Evolution of the orbit under back-reaction 184
4.1.4 Binaries at cosmological distances 190
4.2 Radiation from rotating rigid bodies 200
4.2.1 GWs from rotation around a principal axis 201
4.2.2 GWs from freely precessing rigid bodies 204
4.3 Radial infall into a black hole 215
4.3.1 Radiation from an infalling point-like mass 215
4.3.2 Tidal disruption of a real star falling into a black hole. Coherent and incoherent radiation 219
4.4 Radiation from accelerated masses 224
4.4.1 GWs produced in elastic collisions 224
4.4.2 Lack of beaming of GWs from accelerated masses 227
4.5 Solved problems 230
4.1. Fourier transform of the chirp signal 230
4.2. Fourier decomposition of elliptic Keplerian motion 233
Further reading 235

5 GW generation by post-Newtonian sources **236**
5.1 The post-Newtonian expansion 237
5.1.1 Slowly moving, weakly self-gravitating sources 237
5.1.2 PN expansion of Einstein equations 239
5.1.3 Newtonian limit 240
5.1.4 The 1PN order 242
5.1.5 Motion of test particles in the PN metric 245
5.1.6 Difficulties of the PN expansion 247
5.1.7 The effect of back-reaction 249
5.2 The relaxed Einstein equations 250
5.3 The Blanchet–Damour approach 253
5.3.1 Post-Minkowskian expansion outside the source 253
5.3.2 PN expansion in the near region 259
5.3.3 Matching of the solutions 263
5.3.4 Radiative fields at infinity 266
5.3.5 Radiation reaction 275
5.4 The DIRE approach 279
5.5 Strong-field sources and the effacement principle 282
5.6 Radiation from inspiraling compact binaries 289
5.6.1 The need for a very high-order computation 290
5.6.2 The 3.5PN equations of motion 292
5.6.3 Energy flux and orbital phase to 3.5PN order 294
5.6.4 The waveform 296
Further reading 299

6 Experimental observation of GW emission in compact binaries **302**
6.1 The Hulse–Taylor binary pulsar 302
6.2 The pulsar timing formula 305
6.2.1 Pulsars as stable clocks 305
6.2.2 Roemer, Shapiro and Einstein time delays 306
6.2.3 Relativistic corrections for binary pulsars 314
6.3 The double pulsar, and more compact binaries 326
Further reading 331

Part II: Gravitational-wave experiments **333**

7 Data analysis techniques **335**
7.1 The noise spectral density 335
7.2 Pattern functions and angular sensitivity 339
7.3 Matched filtering 343
7.4 Probability and statistics 346
7.4.1 Frequentist and Bayesian approaches 346
7.4.2 Parameters estimation 350
7.4.3 Matched filtering statistics 356
7.5 Bursts 361
7.5.1 Optimal signal-to-noise ratio 361

7.5.2 Time–frequency analysis 365
7.5.3 Coincidences 369
7.6 Periodic sources 371
7.6.1 Amplitude modulation 373
7.6.2 Doppler shift and phase modulation 375
7.6.3 Efficient search algorithms 381
7.7 Coalescence of compact binaries 387
7.7.1 Elimination of extrinsic variables 388
7.7.2 The sight distance to coalescing binaries 390
7.8 Stochastic backgrounds 392
7.8.1 Characterization of stochastic backgrounds 393
7.8.2 SNR for single detectors 397
7.8.3 Two-detector correlation 400
Further reading 413

8 Resonant-mass detectors 415
8.1 The interaction of GWs with an elastic body 415
8.1.1 The response to bursts 415
8.1.2 The response to periodic signals 420
8.1.3 The absorption cross-section 421
8.2 The read-out system: how to measure extremely small displacements 427
8.2.1 The double oscillator 428
8.2.2 Resonant transducers 432
8.3 Noise sources 436
8.3.1 Thermal noise 437
8.3.2 Read-out noise and effective temperature 443
8.3.3 Back-action noise and the quantum limit 446
8.3.4 Quantum non-demolition measurements 449
8.3.5 Experimental sensitivities 453
8.4 Resonant spheres 459
8.4.1 The interaction of a sphere with GWs 459
8.4.2 Spheres as multi-mode detectors 466
Further reading 469

9 Interferometers 470
9.1 A simple Michelson interferometer 470
9.1.1 The interaction with GWs in the TT gauge 471
9.1.2 The interaction in the proper detector frame 476
9.2 Interferometers with Fabry–Perot cavities 480
9.2.1 Electromagnetic fields in a FP cavity 480
9.2.2 Interaction of a FP cavity with GWs 489
9.2.3 Angular sensitivity and pattern functions 494
9.3 Toward a real GW interferometer 497
9.3.1 Diffraction and Gaussian beams 497
9.3.2 Detection at the dark fringe 504
9.3.3 Basic optical layout 510
9.3.4 Controls and locking 511

9.4 Noise sources 515
9.4.1 Shot noise 516
9.4.2 Radiation pressure 519
9.4.3 The standard quantum limit 522
9.4.4 Displacement noise 524
9.5 Existing and planned detectors 528
9.5.1 Initial interferometers 528
9.5.2 Advanced interferometers 532
Further reading 535

Bibliography **537**

Index **549**