

Contents

<i>Preface</i>	<i>page xi</i>
<i>Acknowledgements</i>	xiii
Introduction	1
Part I BKL Analysis	5
1 Basic Structure of Cosmological Singularity	7
1.1 Synchronous Reference System	7
1.2 The Gravitational Field Equations	9
1.3 General Solution	10
1.4 Definition of Cosmological Singularity	11
1.5 Kasner-Like Singularities of Power Law Asymptotics	13
1.6 Instability of Kasner Dynamics	17
1.7 Transition to the New Regime	20
1.8 Oscillatory Nature of the Generic Singularity	24
1.9 Rotation of Kasner Axes	27
1.10 Final Comments	30
2 Homogeneous Cosmological Models	32
2.1 Homogeneous Models of Bianchi Types IX and VIII	32
2.2 Equations of Motion for Homogeneous Models	33
2.3 Models of Types IX and VIII with Fixed Kasner Axes	38
2.4 Models of Types IX and VIII with Rotating Axes	41
2.5 On the Extension to the Inhomogeneous Case	46
3 On the Cosmological Chaos	48
3.1 Stochasticity of the Oscillatory Regime	48
3.2 Historical Remarks	51
3.3 Gravitational Turbulence	53

4	On the Influence of Matter and Space-Time Dimension	57
4.1	Introduction	57
4.2	Perfect Fluid	61
4.3	Perfect Fluid of Stiff Matter Equation of State	65
4.4	Yang–Mills and Electromagnetic Fields	71
4.5	Scalar Field	80
4.6	Pure Gravity in Higher Dimensions	81
4.7	Generalized Kasner Solutions: Rigorous Results	87
4.8	On the Influence of Viscous Matter	88
 Part II Cosmological Billiards		95
5	The Billiard of Four-Dimensional Vacuum Gravity	97
5.1	Hamiltonian Form of the Action	97
5.2	Supermetric	99
5.3	More on Hyperbolic Space \mathbb{H}_2	104
5.4	Kasner Solution Revisited	109
5.5	Hamiltonian in Pseudo-Gaussian Gauge	116
5.6	BKL Limit and Emergence of Billiard Description	120
5.7	Collision Law	127
5.8	Miscellanea	132
5.9	Chaos and Volume of the Billiard Table	136
5.10	Coxeter Group for Pure Gravity in Four Dimensions	138
6	General Cosmological Billiards	145
6.1	Models – Hamiltonian Form of the Action	145
6.2	Geometry of the Space of Scale Factors	148
6.3	Hyperbolic Space in M Dimensions	154
6.4	Hamiltonian in Iwasawa Variables and BKL Limit	158
6.5	Walls	161
6.6	Chapter 4 Revisited	165
6.7	Miscellanea	167
7	Hyperbolic Coxeter Groups	170
7.1	Introduction	170
7.2	Convex Polyhedra in Hyperbolic Space	171
7.3	Coxeter Groups: General Considerations	174
7.4	Coxeter Groups: Examples	176
7.5	Coxeter Groups and Weyl Groups	182
7.6	Coxeter Groups Associated with Gravitational Theories	188
7.7	The Kac–Moody Symmetry Conjecture	198

Appendices	201
A Various Technical Derivations	203
A.1 Perturbations to Kasner-Like Asymptotics	203
A.2 Frame Components of the Ricci Tensor	205
A.3 Exact Solution for Transition Between Two Kasner Epochs	206
A.4 The Derivation of the Rotation Effect of the Kasner Axes	208
B Homogeneous Spaces and Bianchi Classification	213
B.1 Homogeneous Three-Dimensional Spaces	213
B.2 Bianchi Classification	215
B.3 Frame Vectors	217
B.4 On the Freezing Effect in Bianchi IX Model	219
C Spinor Field	221
C.1 Equations of the Gravitational and Spinor Fields	222
C.2 An Exact Homogeneous Solution for the Massless Case	224
C.3 The General Solution in the Vicinity of the Singularity	228
D Lorentzian Kac-Moody Algebras	231
D.1 Definitions	231
D.2 Roots	233
D.3 The Chevalley Involution	234
D.4 Three Examples	234
D.5 The Affine Case	236
D.6 The Invariant Bilinear Form	237
D.7 The Weyl Group	240
D.8 Hyperbolic Kac-Moody Algebras	243
D.9 Overextensions of Finite-Dimensional Lie Algebras	247
<i>References</i>	252
<i>Index</i>	261