

# Contents

*Preface*

*page xi*

	<b>PART ONE THEORY</b>	1
<b>1</b>	<b>Analytical Methods</b>	3
	1.1 Setting and basic terminology	3
	1.2 Center manifold reduction	6
	1.3 Normal forms	8
	1.4 Approximating ODEs	10
	1.5 Simplest bifurcations of planar ODEs	11
	1.6 Pontryagin–Melnikov theory	25
<b>2</b>	<b>One-Parameter Bifurcations of Maps</b>	30
	2.1 Codim 1 bifurcations of fixed points and cycles	30
	2.2 Some global codim 1 bifurcations	43
<b>3</b>	<b>Two-Parameter Local Bifurcations of Maps</b>	50
	3.1 Cusp and generalized period-doubling bifurcations	51
	3.2 CH (Chenciner bifurcation)	54
	3.3 Strong resonances	61
	3.4 Fold–flip and fold–Neimark–Sacker bifurcations	87
	3.5 Flip–Neimark–Sacker and double Neimark–Sacker bifurcations	106
	3.6 Historical perspective	132
	Appendices	134
<b>4</b>	<b>Center Manifold Reduction for Local Bifurcations</b>	185
	4.1 The homological equation and its solutions	186
	4.2 Critical normal form coefficients for local codim 2 bifurcations	190

4.3	Branch switching at local codim 2 bifurcations	204
	Appendix: Fifth-order coefficients for flip–Neimark–Sacker and double Neimark–Sacker	210
	<b>PART TWO SOFTWARE</b>	217
5	<b>Numerical Methods and Algorithms</b>	219
5.1	Continuation of cycles	219
5.2	Continuation of codimension 1 bifurcation curves	220
5.3	Computation of normal form coefficients	224
5.4	Computation of one-dimensional invariant manifolds of saddle fixed points	229
5.5	Continuation of connecting orbits	232
5.6	Bifurcations of homoclinic orbits	238
5.7	Computation of Lyapunov exponents	241
6	<b>Features and Functionality of MATCONTM</b>	243
6.1	General description of MATCONTM	244
6.2	The mapfile	248
6.3	Numerical continuation	250
6.4	Calling the Continuer	254
7	<b>MATCONTM Tutorials</b>	258
7.1	Tutorial 1: iteration of maps and continuation of fixed points and cycles	258
7.2	Tutorial 2: two-parameter local bifurcation analysis	274
7.3	Tutorial 2: invariant manifolds and connecting orbits	294
7.4	Tutorial 4: computation of Lyapunov exponents	308
	<b>PART THREE APPLICATIONS</b>	319
8	<b>The Generalized Hénon Map</b>	321
8.1	Introduction	321
8.2	Homoclinic bifurcations and GHM	324
8.3	Bifurcation diagrams of GHM	329
8.4	Interpretation	348
8.5	Discussion	351
9	<b>Adaptive Control Map</b>	354
9.1	Local bifurcations	354
9.2	Numerical continuation	357
9.3	Derivatives for the adaptive control map	358

<b>10</b>	<b>Duopoly Model of Kopel</b>	362
	10.1 Description of the model	362
	10.2 Fixed points and codim 1 bifurcations	363
	10.3 Normal forms of codim 1 bifurcations	365
	10.4 Codim 2 bifurcations	367
	10.5 Codim 2 normal form coefficients	370
	10.6 Numerical analysis using MATCONTM	372
	10.7 Conclusions	382
<b>11</b>	<b>The SEIR Epidemic Model</b>	385
	11.1 The model	385
	11.2 Bifurcation diagram	386
	<i>References</i>	389
	<i>Index</i>	400