Preface

Real-Life Optimization Problems and Parameter Space Investigation Method

What Is This Book About?

While searching for optimal solutions, two questions arise: where to search and how to search. Various optimization methods, to which an uncountable set of works is devoted, answer the second question. However, without correctly answering the first question, the search for optimal solutions can lead to unsatisfactory results. Usually such "optimization" is observed when solving real-life problems. Notice that, with the rare exception, very little attention is paid to a fundamental problem such as the determination of the feasible solution set or where to search for optimal solutions. Defining this set is directly related to the correct statement of the optimization problem. This usually represents the most significant difficulties for the expert. Therefore, in the majority of cases, the expert cannot state the problem correctly and ends up solving ill-posed problems. In this book, we will show how to define the feasible solution set and thus answer the fundamental question of where to search for optimal solutions.

Difficulties in Stating an Optimization Problem

The majority of engineering problems (design, identification, design of controlled systems, large-scale systems, predicting from observational data, and so on) are essentially multicriteria. These problems are encountered in all facets of human activity. The expert always wishes to optimize not one important criterion, but all of the most important criteria, many of which are antagonistic. The basic components of a problem statement are constraints on criteria, design variables, and so-called functional dependences. These constraints determine the feasible solution set, a region in the criteria and design variable spaces where optimal solutions should be sought. The determination of the feasible solution set is the essence of problem statement. An important subset of the feasible solution set is referred to as the Pareto optimal solution set. A solution[1] is Pareto optimal if value of one criterion can be improved only at the expense of worsening at least one of the other criteria. In order to solve the optimization problem, one has to identify the Pareto optimal set. Obviously, if the feasible solution set has been determined incorrectly or incompletely, the obtained Pareto solutions may not have practical value. Nowadays this situation is quite typical in the overwhelmingly majority of engineering problems.

Nature of Constraints

Performance criteria (goal functions) (e.g., the fuel consumption, cost, efficiency, and so on) should be optimized. It is desired that, with other things being equal, these criteria would have the extremal (e.g., maximum or minimum) values. Since criteria are contradictory, the definition of criteria constraints represents significant, sometimes insurmountable difficulties [1–5]. Furthermore, there are functional dependences. Unlike criteria, functional dependences do not need to be optimized. It is required that only their respective constraints are satisfied. We recognize two kinds of functional constraints: rigid and "soft" (nonrigid). For example, standards are rigid functional constraints. These constraints are not supposed to be changed—they are known a priori. On the other hand, "soft" functional constraints (e.g., overall dimensions) can be changed. Quite often the correct definition of these constraints is also difficult for the expert. If functional constraints are poorly defined, many interesting solutions become unreasonably unfeasible. As a result, the feasible solution set can be empty.

Functional dependences and criteria depend on design variables (e.g., geometric sizes). Design variables are changed within some boundaries. Quite often these boundaries can be revised, if it leads to the improvement of values of the main criteria.

Notice that in real-life problems, the number of functional and criteria constraints can reach many dozens, if not hundreds, and the dimensionality of design variable vector can reach many hundreds and thousands.

In the traditional statement of optimization problems, constraints are usually given a priori. However, it is unlikely that such constraints are correct, especially given the high dimensionality of the problems and the complexity of

1. Vilfredo Pareto (1848–1923) was an Italian economist. A strong definition of the Pareto optimal set is given in Section 1.1. From now on, we will be using the expression "Compromise solutions" to refer to "Pareto optimal solutions."

the mathematical model. That is why it is necessary to ensure the correctness of given constraints. Otherwise, the optimization can lead to the meaningless results or equally to the loss of important solutions. As mentioned earlier, established optimization methods do not address the problem of defining the feasible solution set.

Taking into account the difficulties of determining constraints, the feasible solution set can be poor or even empty. Therefore, it is very important to help the expert determine the constraints correctly.

Now, let us turn to the example that illustrates the main topic of this book. These days it is impossible to imagine a doctor, even the most gifted one, working without diagnostic tools, such as X-ray, tomography, lab tests, and so on. Likewise, in engineering problems, it is difficult to imagine approaching the challenging tasks without the tools for constructing and analyzing the feasible solution set. The tools that an expert should use to state and solve real-life problems are discussed in our book.

The Parameter Space Investigation (PSI) Method

In order to construct the feasible solution set, a method called the PSI method has been created and successfully integrated into various fields of industry, science, and technology [1–5]. This method has been used in designing the space shuttle [3, 6–8], nuclear reactors [3], unmanned vehicles, [3, 9], aviation [3, 10–13], cars [3, 14–17], pumping units [18], ships [2, 19–22], metal tools [2, 5, 23], bridges [24], wind power system [25, 26], wireless battlefield networks [27], energy efficient sensor networks [28], and robots [29]. The PSI method is based on the systematic investigation of the multidimensional domain [1–5, 30–36]. A computer generates multidimensional points (each point corresponds to a certain design). This is accomplished by uniformly distributed sequences, nets, and quasi-random points. Then the computer defines the values of criteria in these points. In a continuing dialogue between an expert and a computer, the constraints are repeatedly revised, and, as a result, the feasible and Pareto optimal solutions are determined. Thus, an expert can assess the price of making concessions in various constraints (i.e., what are the losses and the gains). Prior experience has shown that the expert is often ready to change constraints by having information on a sufficient improvement of the values of the main criteria. An expert obtains such information on the basis of the PSI method.

In the PSI method, stating and solving problems is a single process. Such an interactive mode allows us to take into account the experience and knowledge of the expert. As a rule, using the PSI method leads to the correction of the initial problem statement, including the correction of constraints and the mathematical model. From our point of view, all real-life optimization problems have to be stated and solved in an interactive mode.

Sometimes using the PSI method can demand carrying out computationally expensive experiments. In these cases most of computer time is spent on determining the feasible solution set, the correction of the problem statement that eventually leads to obtaining the justified optimal solutions, and there is no way around it.

To the best of our knowledge, the PSI method is the only available method for solving the fundamental problem of constructing the feasible solution set.

The PSI method is implemented in the MOVI (Multicriteria Optimization and Vector Identification) software system [37]. The PSI method and the MOVI software system can be universally applied to many problems and only require access to the mathematical model of the system or object under consideration. Even when a model is not available, the PSI method can still be applied to an approximate mathematical model that can be derived from observational data using statistical machine learning classification and regression algorithms [38–42].

The PSI method and MOVI system provide tools for constructing and analyzing the feasible solution set. First of all, these are the test tables. Other tools are tables of feasible and Pareto optimal solutions, histograms, tables of functional failures, and graphs of criterion versus design variable and criterion versus criterion. All of these tools provide us with unique information about: (1) the distribution of feasible solutions in the design variable and criteria spaces, (2) the work of all constraints, (3) the expediency of their modification, and (4) resources for improvement of the object.

The Number of Criteria

Consider the important issue of the number of criteria in a real-life problem. This number must be no less than necessary. The greater the number of criteria taken into account, the greater the information obtained about: (1) the resources of improving the object (ship, car, nuclear reactor, aircraft, machine tool, robot, submarine), (2) the performance of a mathematical model and constraints, and (3) the accuracy with which the criteria are calculated and how much one can trust them.

The PSI method allows us to consider as many criteria as necessary. For example, in the problems of vector identification, the number of criteria reaches many dozens [5, 28, 29].

Using Single-Criterion Methods to Solve Real-Life Problems

In the overwhelming majority of cases, attempts are made to present real-life multicriteria problems as single-criterion problems. In this case, the expert optimizes only one criterion and imposes constraints on other criteria. This approach is appealing because of the apparent simplicity of solving a complex problem. However, there is something else more important that calls in question the competence of this decision.

Above all, the complex problem of determining the feasible solution set is shifted onto the expert's shoulders. Unfortunately, the expert is usually unable to do this. As a result of substituting a single-criterion problem for a real-life one, we end up with a problem that has little to do with real life. Therefore, the numerous attempts to reduce a multicriteria problem to a single-criterion problem result in "throwing out the baby with the bathwater." The search for optimal solutions without determining the feasible solution set is substituting myth for reality. In other words, there are two alternatives: do it simply, or do it right.

Using single-criterion methods without substantiation of the feasible solution set does not guarantee that the obtained optimal solutions are feasible ones. Furthermore, the expert does not have information about compromise solutions considering all criteria.

Multicriteria Identification

Usually in optimization problems we assume by default that the adequacy of the mathematical model is beyond question (i.e., performance criteria adequately describe the investigated object). However, in the majority of cases it is not true. For this reason multicriteria identification of mathematical models is of fundamental importance in real-life problems. The central point is the construction of the feasible solution set in multicriteria identification problems. These problems are encountered in the production of machine tools, automobiles, ships, and aircraft, where enormous amounts of money are spent on operational development of a prototype. Questions of the multicriteria identification and adequacy of the mathematical model will be discussed in our book.

After the PSI method was developed and used successfully, it became necessary to write a new book where we synthesized the extensive experience of applying the PSI method to a variety of engineering problems. We used it as the basis for a lecture course "Multicriteria Analysis," which is taught in the United States and Russia. The material of the book is set out in a popular, concise form, with a large number of illustrations. The book is intended for a wide circle of readers, from undergraduate to graduate students, to researchers and experts involved in solving applied optimization problems. Reading this book does not require any special mathematical education.

This book is organized as follows. Part I gives an overview of the PSI method and MOVI software system. Specifically, Chapter 1 provides an introduction to multicriteria analysis. Chapter 2 discusses the PSI method that