Preface

The magnetic confinement concept of thermonuclear plasma is one of the areas of controlled fusion research developing in the past 50 years in a quest for clean and unlimited source of energy. The concept is based on the fact that charged particles predominantly follow the magnetic field and therefore the magnetic field localized in a finite area would be able to contain the hot temperature plasma is a spatially bounded area. In magnetic confinement fusion devices, tokamaks and stellarators, the confinement of charged particles has been achieved by the magnetic fields whose field lines lie on the nested (magnetic) toroidal surfaces. However, from the beginning of these studies it was realized that the magnetic field created in real devices may deviate from the intended ideal toroidal configuration due to technical imperfections or plasma instabilities, which eventually may break the symmetry of magnetic field leading to the destruction of magnetic surfaces and plasma confinement. Therefore the problem of stability of the magnetic surfaces with respect to the small deviations (or perturbations) of the magnetic field from the ideal toroidal configuration had become one of the most important issues in the fusion research.

The important fact used in these studies was that a divergence-free magnetic field is equivalent to a Hamiltonian system with $1 + 1/2$ degrees of freedom. Thus the problem of stability of magnetic surfaces has been reduced to the classical problem of the stability of Hamiltonian dynamical systems which has been a subject of many studies in classical and celestial mechanics in the nineteenth and twentieth centuries. Already at that time H. Poincaré noted that the prediction in dynamical systems may become impossible and this problem is not related to the universal laws of motion, but with the specification of the initial conditions, *it may happen that small differences in the initial conditions produce very great ones in the final phenomena. A small error [change] in the former will produce an enormous error [change] in the latter. Prediction becomes impossible* . . . [Poincaré (2009)]. In the modern terminology he discovered the chaotic motion in dynamical systems, now known as a *dynamical chaos*.

Until the 1960s the theory of dynamical systems has been primarily a research area of mathematicians. In the middle of the twentieth century researchers in fusion and accelerator physics revived the interest to this area of classical physics among the physical community whose interest until the 1960s has been mostly occupied by quantum physics. These studies had a crucial impact on the development of the theory of nonlinear dynamics and chaos in physics and beyond it. Particularly, the qualitative criteria of the onset of chaotic motion in physical systems proposed by Chirikov (1959) laid the foundation for the physical theory of chaos.

The phenomenon of chaos of magnetic field lines known as *magnetic stochasticity* (or *magnetic chaos*) plays an important role in the magnetic confinement of fusion plasmas. It concerns with the problems of stability and destruction of the magnetic field surfaces, chaotic behavior of magnetic field lines, and related transport of energy and particles.

In early studies the magnetic stochasticity in fusion devices is considered an undesirable effect which deteriorates the plasma confinement due to the enhanced radial transport of particles and energy along the chaotic field lines. However, at the end of the 1970s it was realized that the phenomenon of magnetic stochasticity can be used to control the transport of energy and particles. Particularly, the *ergodic divertor* concept has been proposed to divert particles and a heat releasing from the plasma to special plates in a controlled way by externally imposed magnetic perturbations which have been implemented in several tokamaks.

At present a magnetic stochasticity has been used that mitigates some undesirable phenomena developing in modern fusion devices. Among them the so-called edge localized modes, known as repetitive heat and particle loading to the divertor targets produced by plasma instabilities at the plasma edge in H-mode regimes. The suppression of these modes is of paramount importance for the International Thermonuclear Experimental Reactor (ITER) to protect wall materials from the damaging effect of huge heat and particles releasing during its operation. The suppression of runaway electrons generated in tokamaks during disruptions of plasma discharges by magnetic perturbations is another example of application of magnetic stochasticity. The runaway electrons with energies up to several tens of MeV may cause severe damage to walls.

These applications show the importance of stochastic magnetic fields to control plasmas in magnetically fusion devices. Therefore, a deeper knowledge of a magnetic stochasticity, its onset, and generic properties would be useful for researchers involved in magnetically fusion studies. There are many textbooks and monographs on chaotic dynamics in which the problems of magnetic stochasticity are presented. However, these presentations are too general and do not take the important features of a magnetic field in fusion devices. At the same time in many books on plasma physics the problem of magnetic stochasticity has been mentioned very briefly and treated only at the elementary level. This description of magnetic field stochasticity became insufficient to describe the present-day experimental observations of very fine patterns of plasma structures related to chaotic magnetic fields. These structures are manifestations of such phenomena as a splitting of separatrices, stable and unstable manifolds, introduced in the mathematical theory of dynamical systems.

The Aim of the Book

This book is intended to fill this gap and to give a systematic theoretical description of magnetic field stochasticity and related charged particle dynamics in magnetically confined fusion devices. The presentation is based on the Hamiltonian formulation of magnetic field lines and charged particles. It employs the classical mathematical tools as well as newly developed methods of Hamiltonian dynamics to study magnetic field lines and particle dynamics in toroidally confined plasmas.

The main intention of this book has been to present generic features of magnetic stochasticity in toroidal plasmas rather than its specific manifestations in a particular fusion device. At present they can be studied by the numerical codes for field line tracing. The knowledge of generic properties of a magnetic stochasticity is much useful to predict and, possible, to effectively control plasma behavior by applied magnetic perturbations. Particularly, we have made the main emphasis on revealing generic and universal features of magnetic perturbations generated by external coils in toroidal plasmas, the properties of chaotic magnetic field lines, and related transport of particles. The asymptotic and mapping methods are intensively employed to describe these generic features of magnetic fields and and their properties.

The choice of materials and the manner of presentation are somehow subjective. The Author tried to cover many issues of a magnetic stochasticity which are well understood. These problems were also studied in tokamaks which are considered the most advanced magnetically confined fusion device. However, some important issues of magnetic stochasticity related to the plasma response to applied magnetic perturbations, the screening of magnetic perturbations, nonlinear transport were outside the scope of the book. At present these problems are not well understood, and they are still under active research.

Structure of the Book

The book consists of 11 chapters and 7 appendices. Chapter 1 presents the Hamiltonian formulation of the equations of magnetic field lines based on the action principle of the classical mechanics. The different forms of the Hamiltonian equations for the magnetic field lines are derived. The magnetic flux coordinates are introduced using the action-angle formalism of classical mechanics. We have considered a model of magnetic field which is used to illustrate the action–angle formalism to study magnetic field lines.

The different analytical models of equilibrium plasmas are presented in Chap. 2. Besides the analytical models for plasmas with circular cross-sections and elongated shapes with magnetic separatrices, we have also described wire models of plasmas. The generic behavior of a magnetic field near the magnetic separatrix and