CONTENTS

Preface xi

Acknowledgements xiii

1 Preliminaries 1

1.1 Introduction 1
1.2 Overview of book 4
1.3 Mathematical preliminaries 6
1.3.1 Vector algebra 7
1.3.2 Stress and strain 10
1.4 Conclusions 11
1.5 References 11

2 Programming 13

2.1 Strategies 13
2.2 FORTRAN 90/95 features 14
2.2.1 Representation of numbers 14
2.2.2 Arrays 15
2.2.3 Array operations 16
2.2.4 Control 19
2.2.5 Subroutines and functions 21
2.2.6 Subprogram libraries and common variables 22
2.3 Charts and pseudo-code 23
2.4 Pre- and postprocessing 24
2.5 Conclusions 25
2.6 Exercises 25
2.7 References 26

3 Discretisation and interpolation 27

3.1 Introduction 27
3.2 One-dimensional elements 28
3.3 Two-dimensional elements 32
3.4 Elements of infinite extent 37
3.5 Subroutines for shape function 39
3.6 Description of physical quantities 40
3.7 Coordinate transformation 42
3.8 Differential geometry 43
3.9 Integration over elements 48
3.9.1 Numerical integration 48
3.10 Program 3.1: Calculation of surface area 51
3.11 Conclusions 53
3.12 Exercises 54
3.13 References 55

4 Material modelling and fundamental solutions 57

4.1 Introduction 57
4.2 Steady state potential problems 58
4.3 Static elasticity problems 64
4.3.1 Constitutive equations 70
4.3.2 Fundamental solutions 73
4.4 Conclusions 82
4.5 References 82

5 Boundary integral equations 83

5.1 Introduction 83
5.2 Trefftz method 84
5.3 Program 5.1: Flow around cylinder, Trefftz method 87
5.3.1 Sample input and output 90
5.4 Direct method 93
5.4.1 Theorem of Betti and integral equations 93
5.4.2 Limiting values of integrals as P coincides with Q 96
5.4.3 Solution of integral equations 99
5.5 Computation of results inside the domain 106
5.6 Program 5.2: Flow around cylinder, direct method 108
5.6.1 Sample input and output 112
5.7 Conclusions 116
5.8 Exercises 117
5.9 References 118

6 Boundary element methods - numerical implementation 119

6.1 Introduction 119
6.2 Discretisation and isoparametric elements 120
6.3 Integration of kernel shape function products 123
6.3.1 Singular integrals and rigid body motions 123
6.3.2 Numerical integration 128

6.3.3 Numerical integration over one-dimensional elements 132
6.3.4 Numerical integration for two-dimensional elements 143
6.4 Conclusions 152
6.5 Exercises 153
6.6 References 154

7 Assembly and solution 155

7.1 Introduction 155
7.2 Assembly of system of equations 156
7.2.1 Symmetry 160
7.2.2 Subroutine MIRROR 165
7.2.3 Subroutine ASSEMBLY 167
7.3 Solution of system of equations 169
7.3.1 Gauss elimination 170
7.3.2 Conjugate gradient solver 173
7.3.3 Scaling 174
7.4 Program 7.1: General purpose program, direct method, one region 174
7.4.1 User's manual 182
7.4.2 Sample input file 184
7.5 Conclusions 185
7.6 Exercises 186
7.7 References 188

8 Postprocessing 189

8.1 Introduction 189
8.2 Computation of boundary results 190
8.2.1 Potential problems 190
8.2.2 Elasticity problems 194
8.3 Computation of internal results 199
8.3.1 Potential problems 200
8.3.2 Elasticity problems 203
8.4 Program 8.1: Postprocessor 209
8.4.1 Input specification 216
8.5 Conclusions 216
8.6 Exercises 216
8.7 References 217

9 Test examples 219

9.1 Introduction 219
9.2 Cantilever beam 220
9.2.1 Problem statement 220
9.2.2 Boundary element discretisation and input 220
9.2.3 Results 222
9.2.4 Comparison with FEM 227
9.2.5 Conclusions 227
9.3 Circular excavation in infinite domain 228

9.3.1 Problem statement 228
9.3.2 Boundary element discretisation and input 229
9.3.3 Results 231
9.3.4 Comparison with FEM 232
9.3.5 Conclusions 233
9.4 Square excavation in infinite elastic space 234
9.4.1 Problem statement 234
9.4.2 Boundary element discretisation and input 234
9.4.3 'Quarter point' elements 237
9.4.4 Comparison with finite elements 239
9.4.5 Conclusions 240
9.5 Spherical excavation 240
9.5.1 Problem statement 241
9.5.2 Boundary element discretisation and input 241
9.5.3 Results 245
9.5.4 Comparison with FEM 245
9.6 Conclusions 245
9.7 References 246

10 Multiple regions 247

10.1 Introduction 247
10.2 Multi-region assembly 248
10.3 Stiffness matrix assembly 252
10.3.1 Partially coupled problems 255
10.3.2 Example 257
10.4 Computer implementation 262
10.4.1 Subroutine stiffness_BEM 263
10.5 Program 10.1: General purpose program, direct method, multiple regions 269
10.5.1 User's manual 279
10.5.2 Sample problem 281
10.6 Conclusions 284
10.7 References 285

11 Edges and corners 287

11.1 Introduction 287
11.2 Potential problems 288
11.3 Two-dimensional elasticity 290
11.3.1 Region assembly with corners 296
11.4 Three-dimensional elasticity 301
11.5 Implementation 304
11.1.1 Subroutine for detecting corners 305
11.1.2 Subroutine for computing auxiliary equation coefficients 307
11.6 Conclusions 309
11.7 References 310

12 Body Forces 311

12.1 Introduction 311
12.2 Gravity 312
12.2.1 Postprocessing 314
12.3 Initial strains 317
12.3.1 Volume cells 320
12.3.2 Numerical evaluation of volume integrals 322
12.3.3 Postprocessing 323
12.4 Initial stresses 326
12.4.1 Numerical evaluation of volume integrals 329
12.4.2 Postprocessing 330
12.5 Implementation 333
12.6 Example 333
12.7 Conclusions 335
12.8 References 336

13 Non-linear problems 337

13.1 Introduction 337
13.2 General solution procedure 338
13.3 Plasticity 339
13.3.1 Elastoplasticity 339
13.3.2 Viscoplasticity 342
13.3.3 Method of solution 343
13.3.4 Evaluation of singular integrals 345
13.3.5 Computation of internal stresses 347
13.3.6 Example 349
13.4 Contact problems 351
13.4.1 Method of analysis 352
13.4.2 Solution procedure 355
13.4.3 Example of application 356
13.5 Conclusions 358
13.6 References 358

14 Coupled boundary element/finite element analysis 359

14.1 Introduction 359
14.2 Coupling theory 360
14.2.1 Coupling to finite elements 360
14.2.2 Coupling to boundary elements 367
14.3 Examples 367
14.4 Conclusion 370
14.5 References 370

15 Industrial applications 373

15.1 Introduction 373
15.2 Mechanical engineering 375
15.2.1 A cracked extrusion press causes concern 375
15.3 Geotechnical engineering 379

15.3.1 Instability of slope threatens village 379
15.3.2 Analysis of tunnel advance in anisotropic rock 382
15.3.3 Tunnel approaching fault 384
15.3.4 CERN caverns 386
15.4 Geological engineering 390
15.4.1 How to find gold with boundary elements 390
15.5 Civil engineering 393
15.5.1 Arch dam 393
15.6 Conclusions 395
15.7 References 395

Appendix A: Program libraries 397

A.1 UTILITY_LIB 397
A.2 GEOMETRY_LIB 409
A.3 INTEGRATION_LIB 416
A.4 ELAST_LIB 421
A.5 LAPLACE_LIB 426
A6 POSTPROC_LIB 428
A7 STIFFNESS_LIB 430

Appendix B: Answers to exercises 437

Index 453