# Contents


Preface *page* xiii
Acknowledgments xv

**1 Introduction** 1

1.1 Introduction 1
1.2 Brief History of the Field 2
1.3 The Nature of Uncertainties and Probability 4
1.4 Objectives 5
1.5 Software 6
1.6 Organization of Chapters 8

**2 Review of Probability Theory** 11

2.1 Introduction 11
2.2 Elements of Set Theory 11
2.3 Basic Rules of Probability Theory 14
2.4 Random Variable 20
2.5 Reliability and Hazard Functions 24
2.6 Multiple Random Variables 26
2.7 Expectation and Moments 31
2.8 Distribution of Functions of Random Variables 34
2.9 Second Moments of Functions of Random Variables 38
2.10 Extreme-Value Distributions 42
2.11 Probability Distribution Models 49
Appendix 2A. Probability Distribution Models 50
Problems 54

**3 Multivariate Distributions** 58

3.1 Introduction 58
3.2 The Multinormal Distribution 58
3.3 The Multivariate Lognormal Distribution 62
3.4 Joint Distribution as Product of Conditionals 65

3.5 Multivariate Distributions with Prescribed Marginals 68
3.5.1 Morgenstern Family of Multivariate Distributions 68
3.5.2 Nataf Family of Multivariate Distributions 72
3.5.3 Copula Distributions 76
3.6 Transformation to the Standard Normal Space 79
3.6.1 Single Random Variable 80
3.6.2 Statistically Independent Random Variables 81
3.6.3 Multinormal Random Variables 82
3.6.4 Nataf-Distributed Random Variables 83
3.6.5 General Non-Normal Random Variables 84
Appendix 3A. Correlation Coefficients for Nataf Distribution 87
Appendix 3B. Brief on Cholesky Decomposition 90
Problems 91

**4 Formulation of Structural Reliability**

4.1 Introduction 93
4.2 The *R-S* Reliability Problem 93
4.2.1 Solution by Conditioning on $S$ 93
4.2.2 Solution by Conditioning on $R$ 94
4.2.3 Formulation in Terms of Safety Margin 95
4.2.4 Formulation in Terms of Safety Factor 96
4.3 The Tail-Sensitivity Problem 98
4.4 The Generalized Structural Reliability Problem 101
4.5 Concluding Remarks 102
Problems 106
107

**5 Analysis of Structural Reliability Under Incomplete Probability Information** 108

5.1 Introduction 108
5.2 Second-Moment Reliability Methods 109
5.2.1 The Mean-Centered, First-Order, Second-Moment Reliability Method 109
5.2.2 The First-Order, Second-Moment Reliability Method 111
5.2.3 Algorithm for Finding the Design Point 114
5.2.4 The Generalized (Second-Moment) Reliability Index 121
5.3 Reliability Methods with Beyond Second-Moment Information 123
5.3.1 Knowledge of Third and Fourth Moments 124
5.3.2 Knowledge of Marginal Distributions 127
5.3.3 Reliability Index Based on Upper Chebyshev Bound 130
5.4 Concluding Remarks 131
Problems 133

**6 The First-Order Reliability Method** 135

6.1 Introduction 135
6.2 Properties of the Standard Normal Space 136
6.3 The First-Order Reliability Method 139
6.4 Accuracy of the FORM Approximation 149
6.5 FORM Measures of Importance of Random Variables 153
6.6 FORM Parameter Sensitivities 158
6.7 Sensitivities with Respect to Alternative Set of Parameters 163
6.8 Importance Vectors with Respect to Means and Standard Deviations 165
6.9 Multiple Design Points 167
6.10 The Inverse Reliability Problem 172
6.11 FORM Approximation of the CDF and PDF of a Function of Random Variables 175
6.12 Concluding Remarks 177
Problems 179

**7 The Second-Order Reliability Method** 185

7.1 Introduction 185
7.2 Classical Formulation of the Second-Order Reliability Method 185
7.3 Gradient-Based SORM 189
7.4 Point-Fitting SORM 193
7.5 Concluding Remarks 197
Problems 198

**8 System Reliability** 200

8.1 Introduction 200
8.2 Representation of Systems 201
8.3 Definition of System Reliability 206
8.4 System Reliability by Expectation 206
8.5 System Reliability by the Inclusion–Exclusion Rule 209
8.6 Bounds on Series-System Reliability 212
8.7 Bounds on System Reliability by Linear Programming 215
8.8 Matrix-Based System Reliability Method 221
8.9 Formulation of Structural System Reliability 224
8.10 First-Order Approximations for Series and Parallel Systems 225
8.11 Bi-Component Bounds for FORM Approximation of Series Systems 232
8.12 Event-Tree Approach for Modeling Sequential Failures 238
8.13 Component Importance Measures 245

8.14 System Sensitivity Measures 252
8.15 Concluding Remarks 256
Problems 258

**9 Simulation Methods** 262

9.1 Introduction 262
9.2 Generation of Pseudorandom Numbers with Uniform Distribution 263
9.3 Generation of Pseudorandom Numbers with Specified Distribution 264
9.4 Generation of Pseudorandom Numbers for Dependent Random Variables 265
9.5 Monte Carlo Simulation 266
9.6 Use of Antithetic Variates 270
9.7 Importance Sampling 271
9.8 Numerical Integration by Importance Sampling 279
9.9 Directional Sampling 283
9.10 Orthogonal-Plane Sampling 288
9.11 Subset Simulation 290
9.12 Reliability Sensitivities by Simulation 294
9.13 Concluding Remarks 298
Problems 300

**10 Bayesian Parameter Estimation and Reliability Updating** 301

10.1 Introduction 301
10.2 Sources and Types of Uncertainties 302
10.3 Bayesian Parameter Estimation 304
10.3.1 Formulation of the Likelihood Function 306
10.3.2 Selection of Prior Distribution 309
10.3.3 Conjugate Priors for the Normal Distribution 313
10.4 Assessing Mathematical Models of Physical Phenomena 319
10.4.1 Formulation of Likelihood Function for Model Assessment 320
10.5 Analysis of Structural Reliability under Statistical and Model Uncertainties 327
10.6 Updating of Structural Reliability 340
10.7 Updating the Distribution of Basic Random Variables 346
10.8 Concluding Remarks 348
Appendix 10A. Conjugate-Pair Distributions 350
Problems 354

**11 Time- and Space-Variant Reliability Analysis** 357

11.1 Introduction 357
11.2 Review of Random Processes 361

11.3 Power-Spectral Density of a Stationary Process 366
11.4 The Gaussian Process 369
11.5 Solution Approaches for Reliability Analysis 370
11.5.1 Upper-Bound Solution 370
11.5.2 Lower-Bound Solution 378
11.6 The Poisson Process 380
11.6.1 Poisson Process with Random Selections 385
11.6.2 Waiting and Interarrival Times in a Poisson Process 385
11.6.3 Poisson Approximation for Time- and Space-Variant Reliability Problems 387
11.7 Stochastic Load Models and Load Combination 388
11.7.1 Ferry Borges-Castanheta Load Model 389
11.7.2 The Filtered Poisson Model 391
11.7.3 The Poisson Square-Wave Process 393
11.7.4 The Poisson Pulse Process 394
11.8 Combination of Homogeneous Poisson Pulse Load Processes 395
11.9 Concluding Remarks 401
Appendix 11A. Derivation of Limit Formula for Mean Down-Crossing Rate 402
Problems 403

**12 Finite-Element Reliability Methods** 405

12.1 Introduction 405
12.2 Brief Review of the Finite-Element Formulation 407
12.3 Formulation of the Finite-Element Reliability Problem 409
12.4 The Direct-Differentiation Method 413
12.5 Discrete Representation of Random Fields 425
12.6 The Spectral Stochastic Finite-Element Method 435
12.7 Response-Surface Methods 437
12.8 Concluding Remarks 444

**13 Reliability Methods for Stochastic Structural Dynamics** 446

13.1 Introduction 446
13.2 Discrete Representation of Random Processes 447
13.3 Response of Linear System to Gaussian Excitation 452
13.4 Response of Linear System to Non-Gaussian Excitation 455
13.5 Tail-Equivalent Linearization for Nonlinear Stochastic Dynamic Analysis 459
13.5.1 Properties of the Tail-Equivalent Linear System 464
13.6 Level Crossings of the Response Process 469
13.7 The First-Passage Probability 473
13.8 TELM with Multiple Excitations 480

13.9 Evolutionary TELM 486
13.10 Concluding Remarks 490

**14 Reliability-Based Design Optimization** 493

14.1 Introduction 493
14.2 Problem Formulation 497
14.3 Solution by the Decoupling Approach 500
14.3.1 Solution of Problems $P_1$ and $P_{1,sys}$ 500
14.3.2 Solution of Problems $P_2$ and $P_{2,sys}$ 505
14.3.3 Solution of Problems $P_3$ and $P_{3,sys}$ 507
14.4 Sampling-Based RBDO 514
14.5 RBDO Employing Surrogate Models 518
14.6 Buffered Failure Probability Approach 520
14.7 Concluding Remarks 522

**15 Bayesian Network for Reliability Assessment and Updating** 523

15.1 Introduction 523
15.2 Elements of a Bayesian Network 524
15.3 D-Separation Rules 527
15.4 Discretization of Continuous Random Variables 528
15.5 Inference in Bayesian Network 530
15.6 BN Modeling of Components 539
15.7 BN Modeling of Systems 541
15.8 BN Modeling of Random Fields 545
15.9 Dynamic Bayesian Network 556
15.10 Bayesian Network Enhanced by Structural Reliability Methods 561
15.11 Concluding Remarks 571

References 572
Index 587