Contents

1 **Introduction** 1
References 15

2 **Matrices, Vectors, Frames, Transforms** 17
2.1 Matrices 17
2.2 Vectors 21
2.3 Jacobian, Gradient, and Hessian 22
2.4 Orthonormal Bases, Change of Basis 24
2.5 Vectors in Three Dimensions 28
2.6 Some Useful Reference Frames 31
2.6.1 Spacecraft Body Frame 31
2.6.2 Inertial Reference Frames 31
2.6.3 Earth-Centered/Earth-Fixed Frame 32
2.6.4 Local-Vertical/Local-Horizontal Frame 36
2.7 Quaternions 37
2.8 Rotations and Euler's Theorem 40
2.9 Attitude Representations 41
2.9.1 Euler Axis/Angle Representation 41
2.9.2 Rotation Vector Representation 44
2.9.3 Quaternion Representation 45
2.9.4 Rodrigues Parameter Representation 48
2.9.5 Modified Rodrigues Parameters 50
2.9.6 Euler Angles 52
2.10 Attitude Error Representations 59
References 64

3 **Attitude Kinematics and Dynamics** 67
3.1 Attitude Kinematics 68
3.1.1 Attitude Matrix 68
3.1.2 Vector Addition of Angular Velocity 69
3.1.3 Vector Kinematics 70

3.2 Kinematics of Attitude Parameterizations 71
3.2.1 Quaternion Kinematics 71
3.2.2 Rodrigues Parameter Kinematics 72
3.2.3 Modified Rodrigues Parameter Kinematics 72
3.2.4 Rotation Vector Kinematics 72
3.2.5 Euler Angle Kinematics 73
3.2.6 Attitude Error Kinematics 76
3.3 Attitude Dynamics 77
3.3.1 Angular Momentum and Kinetic Energy 77
3.3.2 Rigid Body Dynamics 80
3.3.3 Rigid Body Motion 84
3.3.4 Torque-Free Motion of a Rigid Body 90
3.3.5 Internal Torques 99
3.3.6 External Torques 103
3.3.7 Angular Momentum for Health Monitoring 111
3.3.8 Dynamics of Earth-Pointing Spacecraft 112
References 121

4 Sensors and Actuators 123
4.1 Redundancy 123
4.2 Star Trackers 125
4.2.1 Overview 125
4.2.2 Modes of Operation 126
4.2.3 Field of View, Resolution, Update Rate 127
4.2.4 Star Catalogs 129
4.2.5 Proper Motion, Parallax, and Aberration 130
4.3 Sun Sensors 133
4.4 Horizon Sensors 135
4.5 Magnetometers 135
4.6 Global Positioning System 136
4.6.1 GPS Satellites 138
4.7 Gyroscopes 140
4.7.1 Gyro Measurement Model 143
4.8 Reaction Wheels 147
4.8.1 Reaction Wheel Characteristics 148
4.8.2 Reaction Wheel Disturbances 148
4.8.3 Redundant Wheel Configurations 152
4.9 Control Moment Gyros 166
4.10 Magnetic Torquers 168
4.11 Thrusters 169
4.12 Nutation Dampers 170
References 178

5 Static Attitude Determination Methods 183
5.1 The TRIAD Algorithm 184
5.2 Wahba's Problem 186

5.3 Quaternion Solutions of Wahba's Problem 187
5.3.1 Davenport's q Method 187
5.3.2 Quaternion Estimator (QUEST) 189
5.3.3 Another View of QUEST 191
5.3.4 Method of Sequential Rotations 192
5.3.5 Estimator of the Optimal Quaternion (ESOQ) 194
5.3.6 Second Estimator of the Optimal Quaternion (ESOQ2) 195
5.4 Matrix Solutions of Wahba's Problem 196
5.4.1 Singular Value Decomposition (SVD) Method 196
5.4.2 Fast Optimal Attitude Matrix (FOAM) 198
5.4.3 Wahba's Problem with Two Observations 199
5.5 Error Analysis of Wahba's Problem 201
5.5.1 Attitude Error Vector 201
5.5.2 Covariance Analysis of Wahba's Problem 204
5.5.3 Covariance with Two Observations 207
5.6 MLE for Attitude Determination 208
5.6.1 Fisher Information Matrix for Attitude Determination 212
5.7 Induced Attitude Errors from Orbit Errors 216
5.8 TRMM Attitude Determination 218
5.9 GPS Attitude Determination 223
References 231

6 Filtering for Attitude Estimation and Calibration 235
6.1 Attitude Representations for Kalman Filtering 236
6.1.1 Three-Component Representations 236
6.1.2 Additive Quaternion Representation 237
6.1.3 Multiplicative Quaternion Representation 239
6.2 Attitude Estimation 240
6.2.1 Kalman Filter Formulation 240
6.2.2 Gyro Calibration Kalman Smoother 246
6.2.3 Filtering and the QUEST Measurement Model 254
6.2.4 Mission Mode Kalman Filter 257
6.2.5 Murrell's Version 260
6.3 Farrenkopf's Steady-State Analysis 263
6.4 Magnetometer Calibration 269
6.4.1 Measurement Model 270
6.4.2 Centered Solution 272
6.4.3 The TWOSTEP Algorithm 274
6.4.4 Extended Kalman Filter Approach 276
6.4.5 TRACE Spacecraft Results 277
References 283

7 Attitude Control 287
7.1 Introduction 287
7.2 Attitude Control: Regulation Case 289

7.3 Attitude Control: Tracking Case 294
7.3.1 Alternative Formulation 301
7.4 Attitude Thruster Control 303
7.5 Magnetic Torque Attitude Control 307
7.5.1 Detumbling 308
7.5.2 Momentum Dumping 311
7.6 Effects of Noise 312
7.7 SAMPEX Control Design 318
7.7.1 Attitude Determination 321
7.7.2 Magnetic Torque Control Law 324
7.7.3 Science Modes 324
7.7.4 Reaction Wheel Control Law 332
7.7.5 Simulations 333
References 341

A Quaternion Identities 345
A.1 Cross Product Identities 345
A.2 Basic Quaternion Identities 346
A.3 The Matrices $\Xi(q)$, $\Psi(q)$, $\Omega(\omega)$, and $\Gamma(\omega)$ 349
A.4 Identities Involving the Attitude Matrix 350
A.5 Error Quaternions 354
A.6 Quaternion Kinematics 355
References 359

B Euler Angles 361

C Orbital Dynamics 365
C.1 Geometry of Ellipses 365
C.2 Keplerian Motion 370
C.2.1 Classical Orbital Elements 374
C.2.2 Kepler's Equation 376
C.2.3 Orbit Propagation 379
C.3 Disturbing Forces 382
C.3.1 Non-Spherical Gravity 382
C.3.2 Third-Body Forces 388
C.3.3 Atmospheric Drag 389
C.3.4 Solar Radiation Pressure 390
C.4 Perturbation Methods 392
C.4.1 Variation of Parameters 392
C.4.2 Two Line Elements 393
C.4.3 A Useful Approximation, Secular J_2 Effects Only 395
C.4.4 Sun-Synchronous Orbits 397
C.5 Lagrange Points 398
References 402

D Environment Models 403
D.1 Magnetic Field Models 403
D.1.1 Dipole Model 404
D.2 Atmospheric Density 406
D.2.1 Exponentially Decaying Model Atmosphere 406
D.2.2 Harris-Priester Model Atmosphere 407
D.2.3 Jacchia and GOST Model Atmospheres 408
D.2.4 Jacchia-Bowman 2008 (JB2008) Model Atmosphere 409
D.3 Sun Position, Radiation Pressure, and Eclipse Conditions 420
D.4 Orbital Ephemerides of the Sun, Moon, and Planets 422
References 423

E Review of Control and Estimation Theory 425
E.1 System Modeling 425
E.1.1 Inverted Pendulum Modeling 425
E.1.2 State and Observation Models 428
E.1.3 Discrete-Time Systems 434
E.2 Control Theory 437
E.2.1 Basic Linear Control Design 437
E.2.2 Stability of Nonlinear Dynamic Systems 442
E.2.3 Sliding-Mode Control 444
E.3 Estimation Theory 448
E.3.1 Static-Based and Filter-Based Estimation 449
E.3.2 Batch Least Squares Estimation 451
E.3.3 Sequential Least Squares Estimation 453
E.3.4 Maximum Likelihood Estimation 454
E.3.5 Nonlinear Least Squares 456
E.3.6 Advantages and Disadvantages 462
E.3.7 State Estimation Techniques 462
E.3.8 Linear Covariance Analysis 470
E.3.9 Separation Theorem 470
References 473

F Computer Software 475

Index 477