

FOURIER SERIES AND FOURIER TRANSFORMS

1.1. INTRODUCTION

A Fourier (Jean-Baptiste Joseph Fourier, 1768-1830) series is an approach to represent a periodic time function (a signal or wave) as the sum of an infinite number of sinusoidal function terms. Historically, Fourier introduced his series as a solution for the heat equation in a metal plate. The Fourier series tool is used in several electric engineering applications, vibration analysis, signal and image processing, and many other fields.

Mathematically, the Fourier series is an infinite number of weighted sine and cosine terms (or, equivalently, complex exponential). Classically, it is essential to note that the Fourier series was applied for periodic time functions. Later on, it was applied also for aperiodic functions of a prescribed time interval. In practice, a finite number of terms are taken for periodic signal or wave analysis. Then, although the series is a time function, it contains all information about the fundamental and signals harmonics magnitudes and frequencies. As the terms of the series are increased as the series will approximate the signal more accurately. On the other hand, Fourier transforms service to obtain frequency information directly for both periodic and aperiodic signals.

Primarily, the reader has to know enough information related to the periodic functions and their mathematical definition. It should include how to determine the period and distinguish between odd and even functions. Therefore, this will be the material of the next section.

1.2. MOTIVATION

Consider the time functions shown in Figures 1.1a and 1.1b, which depict periodic square pulses and periodic sawtooth signals of period T , and $2T$, respectively. The reader can quickly note that the shapes of signals are repeated every T , and $2T$ units of time, say seconds. For instance, for the square pulses signal, it starts from the origin